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GEOSTROPHIC EDDIES IN THE OCEAN  
PART I

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Report prepared by: T. Ichiye

Technical Report No. CU-21-65 to the Atomic Energy Commission Contract  
AT (30-1) 2663

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A B S T R A C T

The results of observations of eddies with dimensions of several kilometers to a few hundred kilometers are reviewed. Detailed measurements of an eddy off California revealed the quasi-geostrophic structure of the eddy of intermediate size. Eddies in the eastern half of the Gulf of Mexico are described from hydrographic data collected since 1950. Case histories and dynamical structure of eddies generated by cutoff of increasing meanders of the Gulf Stream and the Kuroshio are discussed. Generation of eddies and perturbations in the ocean due to moving meteorological disturbances are explained from examples in case of extra-tropical cyclones in the ocean polar front of the North Pacific and the one in case of hurricane in the central Gulf of Mexico. Dynamics on development of eddies due to a shearing instability are briefly reviewed.



## 1. INTRODUCTION

Heretofore, most studies on oceanic circulation have dealt only with its averaged condition over either a certain interval of time or a certain domain of space, although observed data were collected at discrete space and time intervals and were interpreted as representative of a finite domain of space and time. Such an approach may be reasonable as a first stage of understanding overall features of the ocean circulation. Also, about a decade ago, it was difficult to obtain data which might reveal detailed synoptic features of the circulation in mid-ocean with a tolerable accuracy. Existence of fluctuations of hydrographic elements both in time and space were only suggested but not confirmed by classical hydrographic techniques.

In order to understand dynamics of the ocean circulation, we have to know the fluctuations of dynamic quantities like velocities, density and pressure as well as their mean values, because in a turbulent fluid system like the ocean there is a substantial amount of energy transfer between the mean motion and the fluctuations. In previous studies, such energy transfer was expressed by eddy viscosity terms which are dependent on the space derivatives of the mean motion and the eddy viscosity. These terms always indicate a dissipation mechanism of the mean motion, that is, energy transfer from the mean motion to the fluctuations. In some modes of motion in the ocean in which energy of the mean motion is dissipated and the rate of dissipation is not large compared with the energy of mean motion, such dissipation mechanism as eddy viscosity may be useful to determine dynamical features of the motion. However, the





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detailed structures of velocity and pressure fields in the western boundary current areas observed either with continuous recordings or with repeated hydrographic transects suggest that the energy is transferred from fluctuations or eddies to the mean current in some areas (Webster, 1962; Ichiye, 1965). This is contrary to statistical theories on turbulence, because they predict energy transfer from larger eddies to smaller ones (Batchelor, 1953) except in a two-dimensional isotropic turbulence field studied by Ogura (1962).

The spectra of turbulence measured in a tidal stream have given strong support to the Kolmogoroff hypothesis which infers the energy transfer from larger to smaller eddies (Grant, Stewart and Moilliet, 1962). These spectra were in the range of wave numbers larger than  $10^{-2} \text{ cm}^{-1}$ . Therefore, these results cannot be applied to eddies of larger scales which contribute to energy transfer among ocean currents. It is necessary to determine the spectrum of such larger eddies with a reasonable accuracy in order to understand energy transfer between the mean motion and its perturbations on an oceanic scale. Such energy transfer plays an important part in maintaining and changing a large scale ocean circulation and yet the Kolmogoroff hypothesis may not be valid in this range of wave numbers. The spectrum of large eddies may be determined by measuring fluctuations of velocity and pressure fields for a sufficiently long period of time, probably with buoy systems. However, it is worthwhile to know behaviors of such eddies which have been observed with rather classical techniques, because such information will be useful for future planning of survey programs utilizing buoy systems or other modern techniques. In Part 1 of this paper, eddies with



linear scales of tens to hundreds kilometers observed in different areas of the ocean are described from various sources of data. In addition, some results of rotating tank experiments are discussed.

The linear dimension of the eddies considered here is of the order of magnitude of  $10^6$  to  $10^7$  cm and the velocity is of 10 cm/sec. The Rossby number  $R_o$  defined as  $U/2Lf$ , where  $U$ ,  $L$  and  $f$  are characteristic speed, length and Coriolis coefficient respectively, becomes  $5 \times 10^{-3}$  to  $5 \times 10^{-2}$ . The Richardson number  $R_i$  defined as  $g(\Delta\rho/\rho)(H/U^2)$ , where  $H$  is the characteristic depth and  $\rho$  and  $\Delta\rho$  are density and its vertical difference, becomes  $10^4$  if we take  $\Delta\rho/\rho = 10^{-3}$  and  $H = 5 \times 10^5$  cm. Therefore, the eddies discussed here are almost in geostrophic equilibrium and correspond to the motion classified as Swallow eddies by Phillips (1964). Thus, these eddies are called geostrophic eddies.

## 2. Geostrophic Eddies Off California

Measurements of an eddy by the CCOFI cruise of October, 1959 off California (Reid and others, 1963) show detailed dynamic structures of the eddy in a quasi-geostrophic equilibrium. The techniques used in this survey were drogue measurements combined with classical hydrographic casts. The data seem to be unique in testing geostrophic relationships in an intermediate scale motion by direct current measurements. In Figure 1 are reproduced some of the results from this survey (Reid and others, 1963). Figures 1A and B represent, respectively the dynamic height anomalies on the 0 and 200-decibar level over the 500-decibar level. Figure 1C indicates the movement of drogues superposed on detailed dynamic topography of the eddy





designated as C. The distributions of tangential velocity are determined from the successive positions of six drogues launched about 9.3 km apart within the eddy, and these are plotted against the distance from the center of the eddy in Figure 2. Since orbits of drogues are approximately elliptical in most cases, the distance is determined from the relation  $\sqrt{(a^2 + b^2) / 2}$  where a and b are semi-axes of the ellipse. The velocity distributions are almost similar to those of a Rankine vortex which has a velocity v proportional to a distance r in an inner region and to  $r^{-1}$  in an outer region (Lamb, 1945), although the observed velocity decreases with the distance more gradually than the inverse law beyond fifty kilometers.

Figure 1 indicates that if an extra station E near the center of the eddy C had not been occupied, the dynamic topography from regular hydrographic stations only would not have revealed the presence of this eddy. The dynamic topography of Figure 1A indicates that six cyclonic eddies with dimensions of about 20 to 30 miles were present beside the eddy C. Considering the rather wide spacings of the regular stations, it can be expected that eddies of such dimensions are numerous, as speculated by Spilhaus (1940). Since these eddies were present in the zone between the offshore southwestward flow and the nearshore northwestward flow, instability of the horizontal shear zone might have been a cause for their generations.

An entire life history of the eddy was not studied by Reid and his collaborators. However, trajectories of the drogues which were tracked from October 13 through October 27 did not change very much during this period. It should be noted that such a small eddy was maintained over two weeks without a substantial change while the eddy of a much larger dimension



(about 100 km by 300 km) observed by Operation Cabot south of the Gulf Stream changed its shape and size in the same period. This may be due to the conditions that shearing instability is much larger along the southern edge of the Gulf Stream than off California and also that the eddy in Operation Cabot was at the stage of generation from the main current of the Gulf Stream.

The process of generation of the eddy C is not known in detail because the only survey before the October cruise was made on August 13th through August 31st. The dynamic topography of 0 and 200 decibars over 500 decibars of this survey is shown in Figures 3A and B, respectively. Comparison of surface dynamic topography of two cruises indicates that two cyclonic eddies designated A and B in the area between  $120^{\circ}$  to  $121^{\circ}$  W and  $32^{\circ}$  to  $33^{\circ}$  N were present through both cruises. The eddy C' whose center was located at about  $31.5^{\circ}$  N and  $119.5^{\circ}$  W during the August cruise seemed to develop into the trough extending southeastwards and into the eddy centered at  $31^{\circ}$  N and  $118^{\circ}$  W. Such development must be checked by more frequent and closely located hydrographic stations. Since the eddy C was rather stationary in the period of one to two weeks and almost in a geostrophic equilibrium, it may be sufficient to take hydrographic data once or twice a week over an area of about 2,000 square miles in order to study the process of development and decay of such eddies. Comparison of dynamic topography of 200 over 500 decibars with that of 0 over 500 decibars of both surveys indicates that most of the eddies with linear scales of 10 to 30 miles present in the surface charts are not recognized in the 200 decibar charts. This suggests





that the disturbances in a pressure field due to such eddies may not reach far below the permanent thermocline. Therefore, observations of eddies of such scales may be done more effectively with bathythermographs or towed temperature and/or salinity sensors than with classical hydrographic casts.

### 3. Eddies in the Gulf of Mexico

Studies of circulation in the Gulf of Mexico by Austin (1955) and Ichiye (1962) indicates that a system of anti-cyclonic eddies with linear dimensions of several hundreds of kilometers always exist in the eastern half of the Gulf. Three examples of surface dynamic topography with a reference level of 1000 decibars are shown in Fig. 4 from the hydrographic surveys obtained in January, 1952 (Alaska Cruise 4-2A) in May-June, 1952 (Alaska Cruise 5-2C) and in February-March, 1962 (HIDALGO Cruise 62-H-3) by Texas A and M College. These three charts represent different patterns of circulation in the eastern section of the Gulf. It is seen that the current entering through Yucatan Strait makes an anti-cyclonic loop in the southeastern part of the Gulf and flows out to the east through Florida Straits. There is always one anti-cyclonic eddy in or north of the Yucatan Strait where the southward countercurrent flows around the eddy near the Cuban side. Cochrane (1963, 1965) determined the annual cycle of the surface current speed at Yucatan Strait from pilot charts, mean sea level differences between Yucatan and Cuba and GEK data and found that the speed is maximum in June and minimum in October and November. The anti-cyclonic eddy near the Yucatan Strait seems to be closely dependent on the speed of the Yucatan Current.



When the Yucatan Current was strong, the center of the eddy was located to the north of the Strait, as in Fig. 4A and C, but when the Current was weak, the center of the eddy was in the eastern part of the Strait, as in Fig. 4B. Cochrane (1963) also found that the core of the Yucatan Current extends on a more or less straight course to the NNE after entering into the Gulf when it is strong, while the core tends to flow eastward north of the Yucatan Shelf when it is weak.

One or two anti-cyclonic eddies were formed to the north of the Yucatan Current as three charts of Fig. 4 indicated. There is no definite relationship between the size and locations of these eddies and the intensity of the Current. Fig. 4C suggested that smaller eddies might have been detected at the northern end of the loop of the Yucatan Current if hydrographic stations with closer distances were occupied. These isolated anti-cyclonic eddies may be generated by cutting off the northward loop of the Yucatan Strait, although there is no observation which showed such process in successive orders. The surface topography obtained through two transects between the Mississippi Delta and Cuba repeated twice during August and September in 1954 indicates that the cutoff anti-cyclonic eddy of about 100 miles stayed at least for a couple of weeks without any substantial change in its location and size (Austin, 1955).

The isotachs obtained from five GEK sections at and north of the Yucatan Strait in May, 1962 (Cochrane, 1963) are plotted in Fig. 5. This figure shows a general tendency that a strong cyclonic shear exists to the left of the maximum current in a narrow band of about ten miles, while a





moderate anti-cyclonic shear exists to the right for a distance of 30 to 40 miles. It also shows that there are low velocity zones 5 to 15 miles to the right of the maximum current. These zones become a notch in a velocity profile across the current and have been noticed in many of the GEK sections across the Florida Straits off Miami and in geostrophic velocity profiles and GEK sections across the Gulf Stream proper east of Cape Hatteras (Stommel, 1958). However, this figure particularly indicates that the low speed zones are not continuous and have a finite length in the direction of the current. It shows that the maximum speed zones are also discontinuous along the current. Such cellular structure of the streamlines may be caused by meander of the current or by eddies imbedded in the current

It is probable that the cellular structure may be related to a shingle structure of surface isotherms at the shoreside edge of the Gulf Stream observed by an airborne infrared thermometer (Von Arx and others, 1955; Ichiye, 1965).

If eddies are a cause for such structure, these eddies may be generated by an orographic effect of Yucatan and Florida Straits, the coastlines of which produce obstacles for the current. This speculation seems to be confirmed by observations of Cochrane (1963) who found that there was no notch in velocity profiles in and north of Yucatan Strait when the current was weak.

In fact, various hydrodynamic experiments show that the flow produces eddies behind an obstacle when the Reynolds number of the current exceeds a critical value (Goldstein, 1938). Results of a model experiment of the circulation in the Gulf of Mexico with a rotating tank (Ichiye and Plutchak, 1963) also confirmed that anti-cyclonic eddies are created by Yucatan





Peninsula and Cuba when the inflow from a nozzle exceeds a certain value.

#### 4. Geostrophic Eddies in the Western Boundary Currents

The primary use of bathythermographs by Spilhaus (1940) was the quick survey of eddies in the neighborhood of the Gulf Stream. He obtained the temperature structure of a cyclonic eddy of about 30 kilometers diameter to the south of the Gulf Stream (Fig. 6). From temperature distribution on the isopycnal surface of  $\sigma_t$  equaling 25, he also found an eddy of diameter of about five kilometers attached to the cyclonic eddy and called it the parasite eddy. He thus distinguished three different sizes of eddies in the Gulf Stream: the largest eddy with a dimension of about 150 kilometers, an intermediate eddy of 30 kilometers and the parasite eddy. Apparently, the largest eddy seemed to be a cyclonic cold eddy generated by meandering of the Gulf Stream and was recognized by upward trend of the permanent thermocline and even of the isotherms below 1000 m deep. The intermediate and parasite eddies seemed to be caused by a shearing instability along the edge of a larger cyclonic eddy, but there has been no further study on these eddies. Operation Cabot in 1950 (Fuglister and Worthington, 1951) informs us of a relatively detailed life history of an eddy of the size comparable to a wave length of meandering, but there is no observation which gives an answer to the question whether intermediate eddies are produced by unstable waves or by breaking of an eddy of a larger size.

During the multiple survey of Gulf Stream'60 (Fuglister, 1964) several isolated eddies of dimensions of 100 to 200 Kilometers were observed with classical hydrographic cast, G E K and transponding drift buoys. These



eddies correspond to the largest size of eddies according to classifications by Spilhaus. Characteristic features of these eddies determined from currents measured directly, dynamic topography and isotherms at 200 m depth of this survey are listed as follows:

<u>No.</u>	<u>Location</u>	<u>Size (km)</u>	<u>Velocity (kt.)</u>	<u>R D</u>	<u>Method</u>
1	36.3°N, 61°W	110	1~2	C	G, H
2	36° N, 64.5° W	220 x 100	<0.5	C	T, H
3	37.5°N, 62.3°W	100	<0.5	C	T, H
4	38.6°N 60°	150 x 100	1~3	C	T, G
5	40.3°N, 51.2°W	100	1~2	A	T, G
6	41.3°N, 63.5 W	100	1~2	A	T, G

where location indicates the position of the center of each eddy, size indicates diameter when it is roughly circular and lengths of long and short axis when it is elongated, velocity indicates a range of tangential velocities near the maximum speed zone, RD indicates direction of rotation of the eddy with C and A for cyclonic and anti-cyclonic, respectively and G, H and T in the last column indicate that the features of the eddy are determined by GEK data, hydrographic data and paths of transpondent buoys, respectively.

Comparison of locations of these eddies with the current axis of the Gulf Stream in this survey indicates that all the cyclonic eddies except the No. 4 eddy and all the anti-cyclonic eddies are located respectively, south and north of the axis. This situation together with a fact that No. 4 eddy was actually in the "sock" or southward trough of the Gulf Stream suggests



that most of the isolated eddies in the Gulf Stream region are generated by cutting off of the meander of the Gulf Stream (Ichiye, 1965).

In the area of the Kuroshio Current, western boundary current of the north Pacific Ocean, eddies of dimensions of 100 to 200 kilometers are abundant north and south of the Kuroshio after it leaves from the coast at a latitude of about  $36^{\circ}$  to  $38^{\circ}$  N. Particularly to the north of the Kuroshio between  $37^{\circ}$  and  $42^{\circ}$  N, there are both anti-cyclonic eddies of warm water and cyclonic eddies of cold water. The warm eddies are the cutoff of the Kuroshio and the cold ones are that of the Oyashio. The latter is the counterpart of the Labrador Current but is stronger and reaches further south than the Labrador Current. Both warm and cold eddies in the area seemed to occasionally have a lifetime of two to three months according to the data obtained in summer to fall, 1948. However, since intervals of the surveys were usually longer than one month, there was a possibility that new eddies might have replaced the old ones during the intervals and have yielded an apparent lifetime longer than a real one (Ichiye, T. and C. Ichiye, 1956). A case history of an anti-cyclonic warm eddy of an extraordinary large scale was studied by use of the hydrographic data obtained in 1954 (Ichiye, 1956). In Fig. 7 a series of surface dynamic topography relative to an 800-decibar level is plotted. The isolated eddy whose center was located at  $38^{\circ}$  N and  $144^{\circ}$  E was first observed in a February-March survey. The only preceding survey was made on September 28th to October 4, 1953 and did not show presence of an eddy, although meander of the Kuroshio extended its crest almost to  $40^{\circ}$  between  $144^{\circ}$  and  $146^{\circ}$  E. The May-June





surface topography of Figure 7 indicates that the eddy was connected with the core and formed a crest of increased meandering. However, the dynamic topography at 100 m depth and below relative to 800 decibars showed a more distinctively isolated pattern of the eddy. Also, temperature distributions at 100 m depth indicate that the eddy was connected with the core during late April to May, but was separated from the core in late May to middle June. The isolated state of the eddy lasted to July as shown in the dynamic topography of June-July in Fig. 7. In the August-September survey, the eddy moved eastwards by about ninety miles and its main part became small and weak, because its northwestern part was split into a spiral-like arm. In the November survey the eddy was completely separated from the core of the Kuroshio and became smaller and weaker than in the preceding survey. The surface tangential velocity  $v$  around the eddy is computed from surface dynamic topography relative to an 800-decibar level in the four surveys and plotted in Fig. 7 against the distance from the center, with an assumption that streamlines in the eddy are concentric circles with areas equal to those surrounded by them. The absolute vorticity  $Z = f - (v/r + \partial v / \partial r)$  is then computed from the tangential velocity and plotted in Fig. 7. It is clearly seen that the tangential velocity and the radius of maximum velocity decreased abruptly in the August-September data. Vorticity distributions indicate that the eddy in August-September and November surveys is more similar to a Rankine vortex, in which the relative vorticity is constant within a certain radius, and vanishes beyond that. The kinetic energy in the upper 100 meter layer of the eddy computed from  $\pi \int v^2 r dr$  using the surface velocity distributions in





Fig. 7 are as follows:

Curve	I	II	III	IV
Kinetic Energy ( $10^{21}$ ergs)	20.6	17.1	3.9	4.1

This table again shows the decrease of kinetic energy in the August-September period. The total kinetic energy of the eddy in the most intense stage may be of the order of  $10^{23}$  ergs if it is assumed that the depth reaches 500 m. The average energy transport of the Kuroshio is  $3.5 \times 10^{22}$  ergs/day (Ichiye, 1955) and it will take about 29 days to form an eddy with total energy of  $10^{23}$  ergs if ten percent of the energy transport of the Kuroshio is used.

#### 5. Eddies Caused by Meteorological Disturbances

There were a few evidences that moving meteorological disturbances like tropical and extra-tropical cyclones caused some changes in the ocean. Particularly when a meteorological disturbance passed near the western boundary current or in the oceanic polar front region, such changes were often manifested as increasing meanders of the current or of the isotherms. Eventually, some of these meanders developed into isolated eddies. In most cases, these eddies belong to the category of the largest eddy discussed in Section 4, but their development from meanders seem to be more rapid than ordinary cutoff of meanders without external disturbing forces. One of those examples which show change of oceanic conditions in the western North Pacific polar front region in 1950 due to a traveling extra-tropical cyclone is shown in Figure 8. Both surface isotherms and vertical temperature sections along  $38^\circ$  N indicate that the cold trough was developed between  $145^\circ$  E



and  $150^{\circ}$  E during and after passage of the cyclone north of the oceanic polar front. Sub-surface structure of the isolated eddy thus produced was known only along the section at  $38^{\circ}$  N. Four examples in 1951 and 1952 studied by Ichiye (1955) showed increasing meanders of surface isotherms of the Kuroshio between  $133^{\circ}$  E and  $140^{\circ}$  E caused by tropical cyclones. These meanders seem to have shorter wave lengths and shorter duration than those in the oceanic polar front described above, although the data of both examples do not have enough accuracy to show precise wave patterns.

The first systematic data concerning a change of oceanic conditions due to a meteorological disturbance are those obtained in the Gulf of Mexico before and after crossing of Hurricane "Hilda" in the period, September 30 to October 4, 1964 (Leipper, 1965). The path of the hurricane, temperature distribution in a vertical section across the path and depths of the  $20^{\circ}$  C isotherm are shown in Figure 9. The temperature data were obtained with BT five to ten days after the passage of the hurricane. A few bathythermograms collected prior to the hurricane showed a well-mixed layer of temperature  $29^{\circ}$  to  $30^{\circ}$  C from the surface to approximately 60 meters depth with a seasonal thermocline below. The temperature data after the hurricane indicate that depth of the mixed layer increased, the mean temperature of the layer increased and that the temperature below the thermocline was much lower near the path of the hurricane eye than outside of the hurricane wind region. Although these data did not clearly indicate that an isolated eddy is generated by passage of the hurricane, the temperature structure below the thermocline was greatly disturbed and the geostrophic circulation was accordingly changed. When this fact is combined with the observations





of meanders of surface isotherms in the polar region discussed above, it is understood that a meteorological disturbance may produce perturbations of the oceanic circulation not only through wind-driven currents but also through thermohaline circulation.

## 6. Dynamic Model on Eddies of Intermediate Sizes

There is no systematic theory on the generation of eddies of geostrophic scales in the ocean. One approach to a problem on development of disturbances in the ocean is to treat hydrodynamic stability of a basic current with a simple velocity distribution when infinitesimal disturbances are superimposed on it. There are several shortcomings in such an approach, in order to avoid mathematical difficulties. Firstly, the geometry and velocity distribution of the basic current must be assumed to be much simpler than in actual ocean currents. Secondly, the disturbances must be assumed to be infinitesimal and sinusoidal in the direction of the basic current, while actual disturbances are of finite dimension and of more complicated configuration. Thirdly, only the initial stage of development of the disturbances can be treated analytically, because in later stages when the disturbances have a finite dimension, non-linear terms become important.

The process of development of an unstable shearing layer into a series of vortices was discussed by Rosenhead (1931), who expressed the vortex sheet at a boundary between two flows with a series of point vortices to the first approximation. However, Birkoff (1962) proved that this approximation has truncation errors. He computed the process of change of the boundary between two flows with a high-speed computer using Rosenhead's





initial condition and also on initial unstable normal mode of small amplitude. He showed that the eddies are rolled up more irregularly than in Rosenhead's analytical result. The results of Rosenhead's and Birkoff's computations are shown schematically in Figure 10, which indicates striking differences between them. Particularly Birkoff's results based on the initial normal mode suggest that the vortex is concentrated in a smaller area and, therefore, has less interaction with other vortices than in Rosenhead's results. This indicates that a vortex due to the unstable boundary may be generated in an isolation and not necessarily in a series, like Karman vortices in a wake. Birkoff also divided the process of development of initially sinusoidal disturbances into three stages: (1) a perturbation stage, during which the amplitudes of perturbation increase as  $A \exp(\alpha t) + B \exp(-\alpha t)$ , (2) a transition stage during which the sinusoidal shape of the perturbation is lost and the amplitude grows more slowly, and (3) an asymptotic change in which a new mode of equilibrium develops. These stages may also occur during development of geostrophic vortices in an unstable shearing current in the ocean, but so far there is no observational evidence nor theoretical study on this problem.

Rigorous analytical treatment of the stability of a current of a large scale was made for a zonal wind similar to the atmospheric westerlies (Phillips, 1964). In these studies it was assumed that the basic current increases linearly with heights and is constant in a horizontal direction in the baroclinic case or it is variable in a lateral direction but is vertically constant in the barotropic case. Eady (1949) and Kuo (1952) discussed a baroclinic instability problem and found that a wave length  $L_c$ , which



yields the maximum growth rate of initially infinitesimal waves is expressed by

$$L_c \approx 2.7 f^{-1} \sqrt{(\Delta \rho / \rho) g H}$$

in which  $H$  is approximately the depth of the basic current,  $f$  is the Coriolis coefficient and  $\Delta \rho$  is the density difference between the upper moving water and the lower still water. When we take  $H = 1 \text{ km}$ ,  $\Delta \rho / \rho = 2 \times 10^{-3}$  and  $f = 10^{-4} \text{ sec}^{-1}$  corresponding to quantities of the western boundary currents, we have  $L_c \approx 120 \text{ km}$ . In the original studies the critical wave length becomes 5000 km for the atmospheric westerlies and this value corresponds to the observed cyclonic waves. If the result of Birkoff (1962) on concentration of vorticity in unstable waves (Fig. 10C) is valid in the rotating system, the waves about 120 km long may produce eddies of diameters of 30 to 50 km, which correspond to the eddies of intermediate sizes, as discussed in Section 4. A numerical computation of development of unstable disturbances in geostrophic currents is a problem to be solved in order to understand generation of eddies in the neighborhood of the ocean currents.

#### A C K N O W L E D G M E N T S

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## Explanation of Figures

- Fig. 1 (A) (B) Dynamic topography (0 over 500 decibars for A and 200 over 500 decibars for B) in October, 1959. The Roman numerals in (A) indicate eddies which were not found on the 200-decibar chart.
- (C) The boxed area of (A) enlarged with the drogue movements indicated by open arrows. The thin lines (two long and one short) indicate the release position of the drogues. "E" means the extra station.
- Fig. 2 Tangential velocity ( $v$ ) obtained by the drogue movements versus the distance from the center of the eddy C. (The curve indicates the velocity in the Rankine vortex.)
- Fig. 3 Dynamic topography (0 over 500 decibar for A and 200 over 500 decibars for B) from CCOFI Cruise 5908 from 13 to 31, August, 1959.
- Fig. 4 Surface dynamic topography relative to the 1000-decibar level in the Gulf of Mexico (A: ALASKA Cruise 4-2 A in January, 1952. B: ALASKA Cruise 5-2 C in May-June, 1952 C: HIDALGO Cruise 62-H-3 in February-March, 1962 from data published by McLellan and Nowlin)
- Fig. 5 Isotachs (in cm/sec) of surface currents measured with GEK by Cochrane (1963) in May, 1962 at and north of Yucatan Strait.
- Fig. 6 Schematic representation of temperature on the isopycnal ( $\sigma_t$  equaling 25) showing an eddy with parasite, after Spilhaus (1940)
- Fig. 7 (A) to (E) Surface dynamic topography relative to an 800-decibar level east of Japan in 1954 (A: February 25 - March 26 B: May 18 - June 17 C: June 24 - July 6 D: August 20 - September 11 E: November 10 - November 30) after Ichiye (1956). (F) Tangential velocities versus distance from the center of the eddy. "A" to "E" indicates curves corresponding to different periods for which dynamic topography are shown. (G) Absolute vorticities versus distance. (The broken horizontal lines (1) and (2) indicate the Coriolis coefficient for the curves "A" to "D" and "E", respectively. The meaning of "A" etc. is explained above.)
- Fig. 8 An example of meanders of an oceanic polar front caused by passage of an extra-tropical cyclone east of Japan in





Explanation of Figures (cont'd.)

December, 1950 after Ichiye (1953). (A) Path of the center of the cyclone and surface temperature in three successive ten-day periods. (B) Two vertical temperature sections along  $38^{\circ}$  N (The first and second sections are based on surveys of the 10th to the 19th and 23rd to the 25th of December, respectively.)

Fig. 9 Change of oceanic conditions due to Hurricane "Hilda" in October, 1964 after Leipper (1965) (A) Hurricane path (dotted line), extent of hurricane winds (broken lines), survey transects (thin full lines) and depths (in meters, thick full lines) of the  $20^{\circ}$  C isotherm after passage of Hurricane "Hilda" (B) Temperature section measured with BT across the path of "Hilda" (The locations of the transects are designated in Figure A).

Fig. 10 Change of a vortex sheet with time, after Birkoff (1962)  
(A) The result of Rosenhead's (1931) approximate calculation.  
(B) The result of numerical calculation based on the same initial condition as Rosenhead .  
(C) The result of numerical calculation based on an initial unstable "normal mode". (T, U and  $\lambda$  indicate time, mean velocity and wave length, respectively.



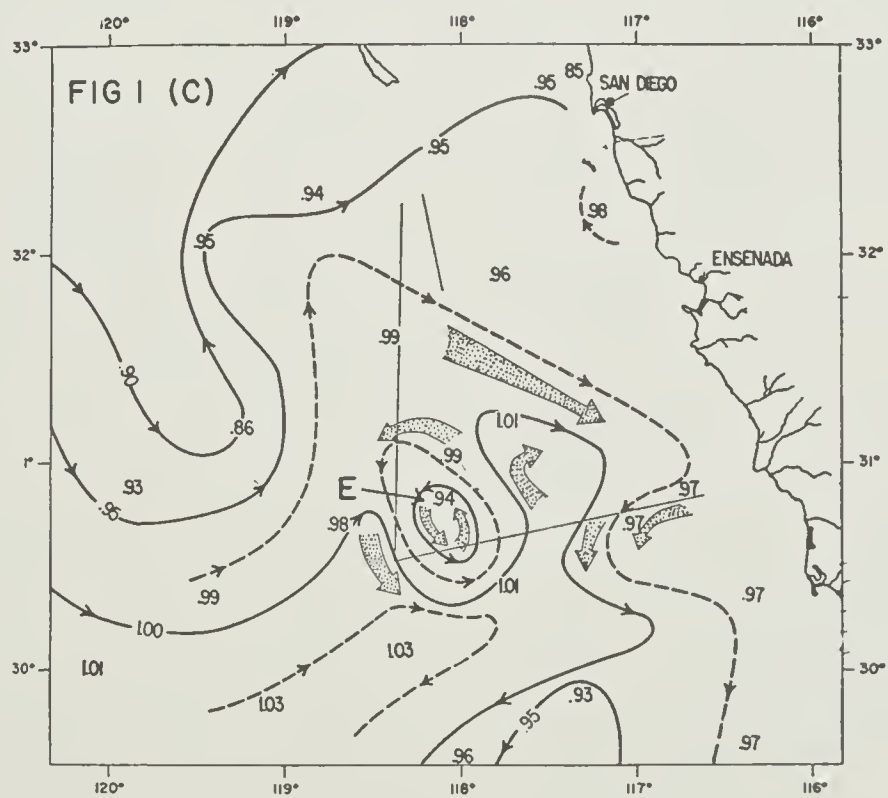
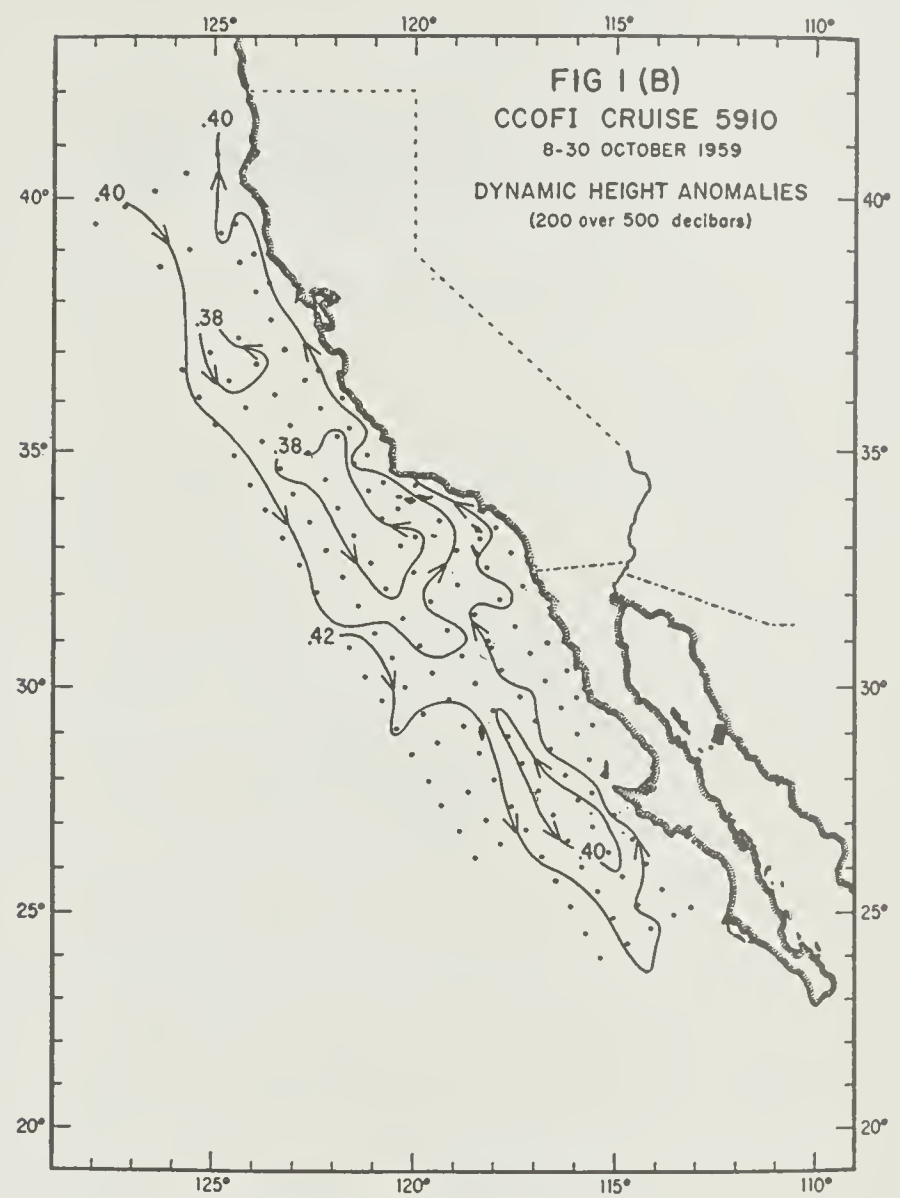
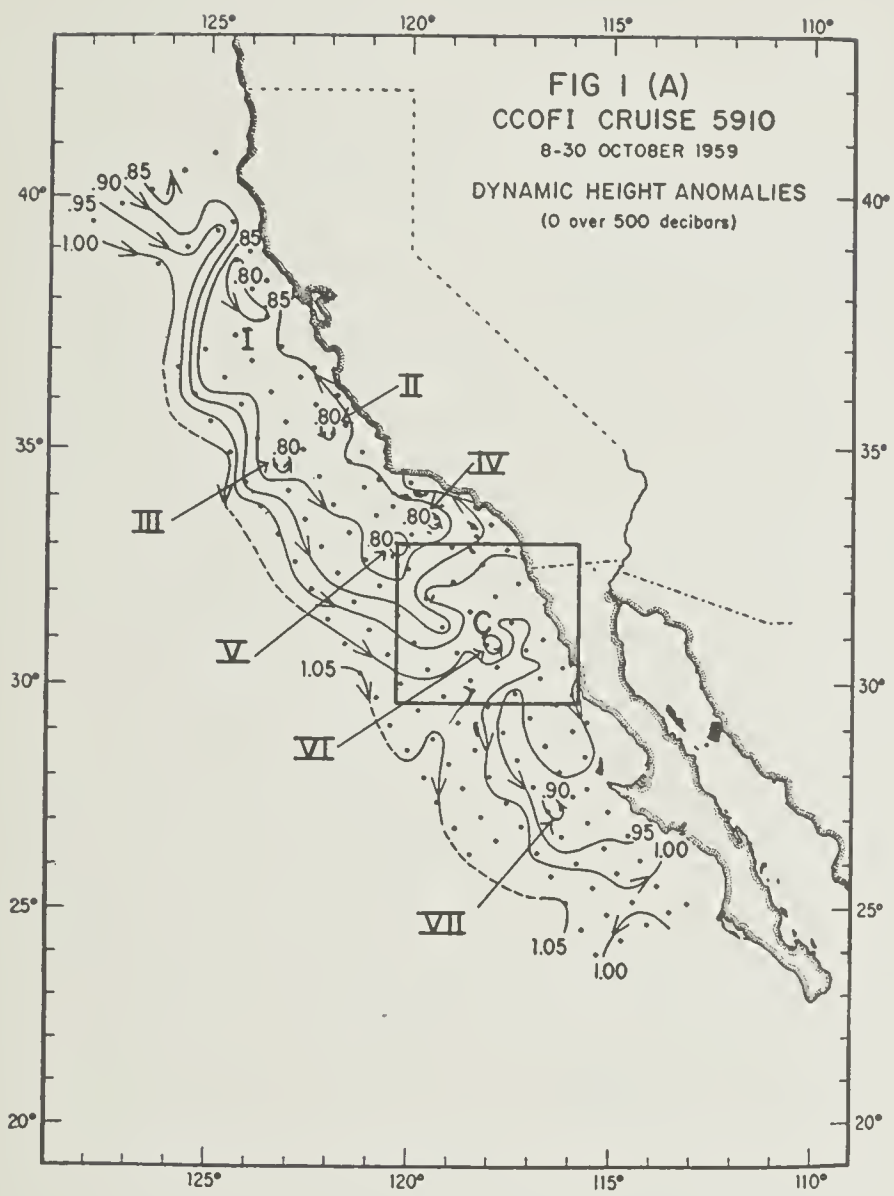






FIG. 2

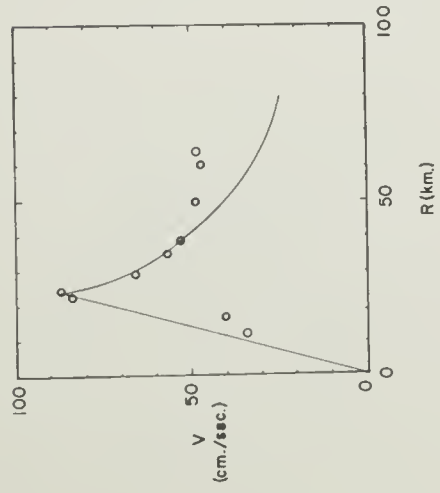


FIG. 3

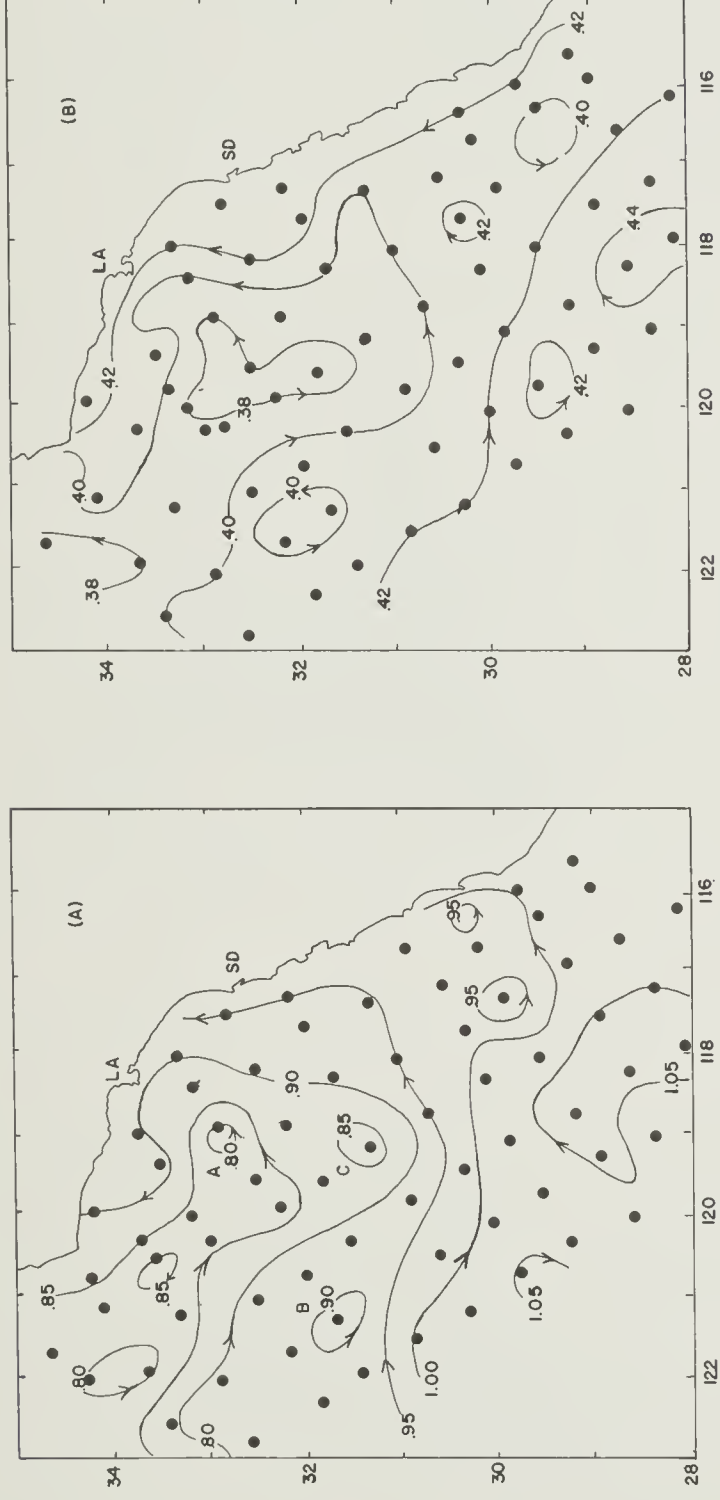
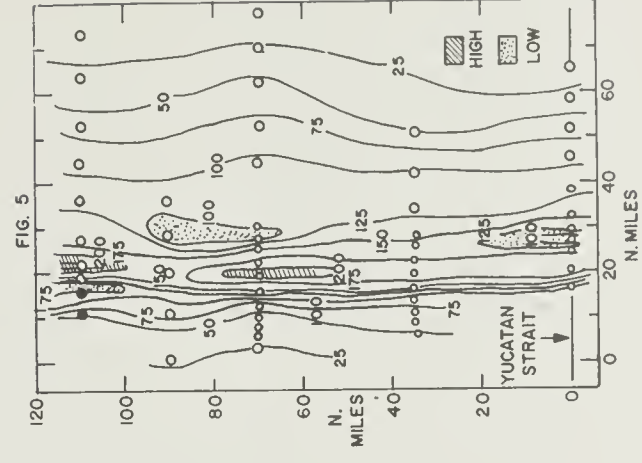
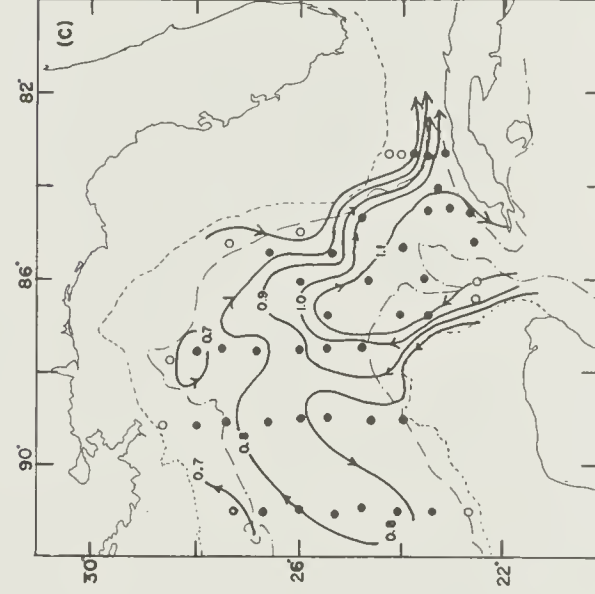
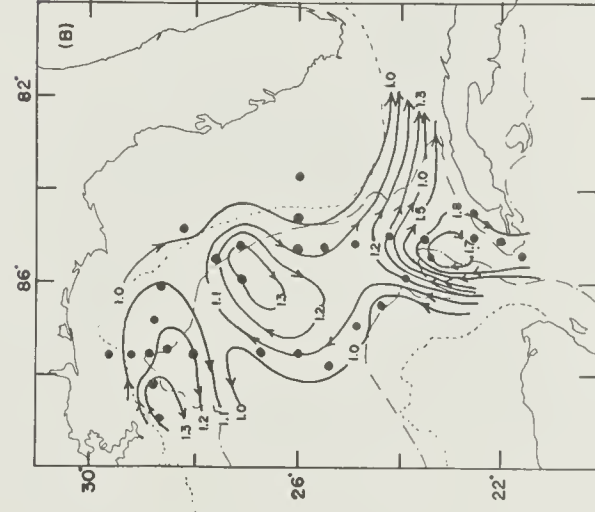
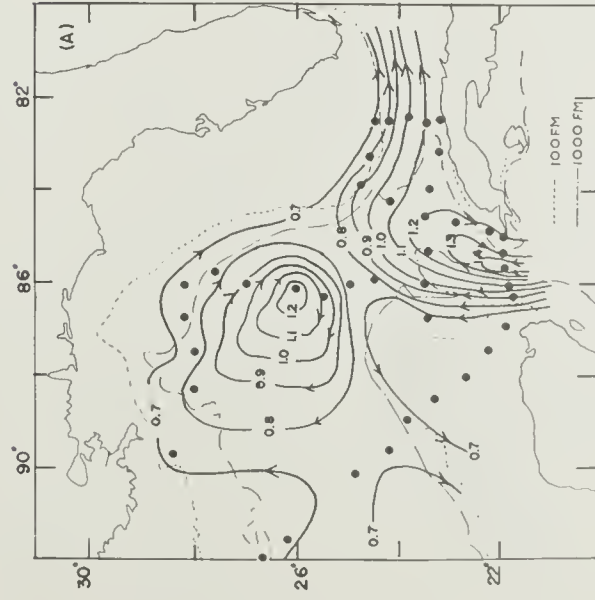


FIGURE 4





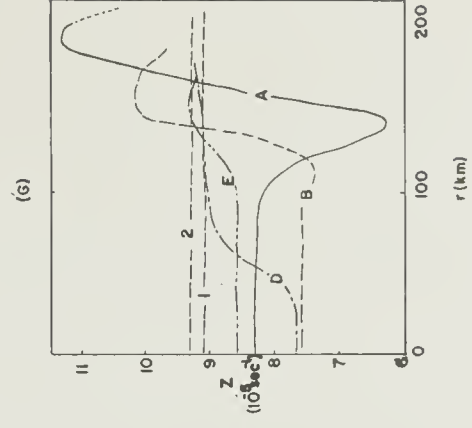
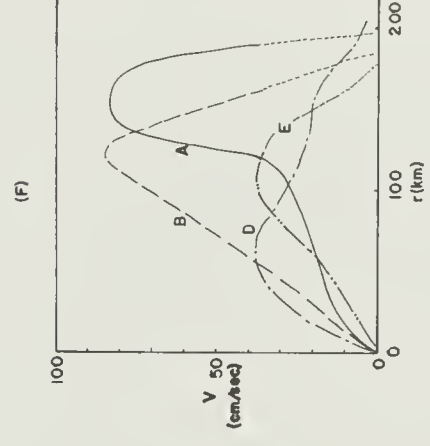
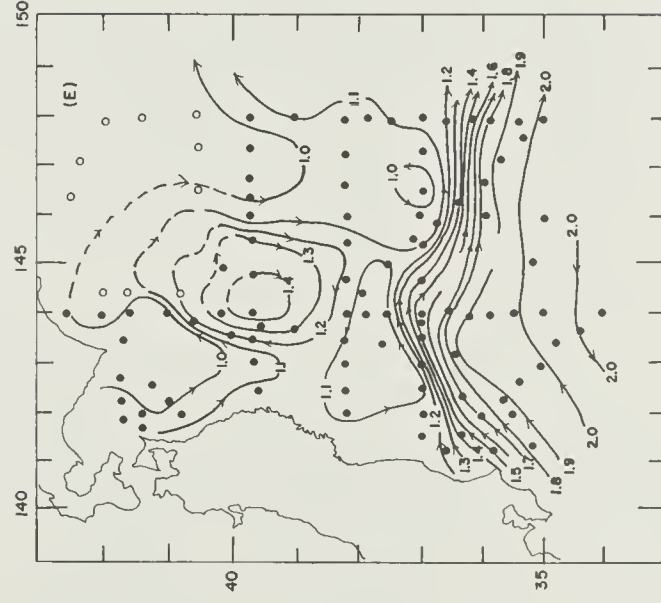
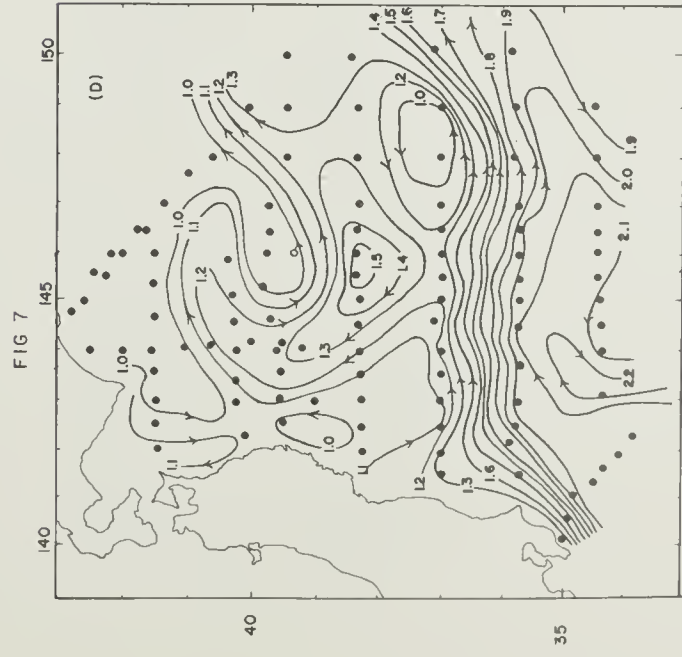
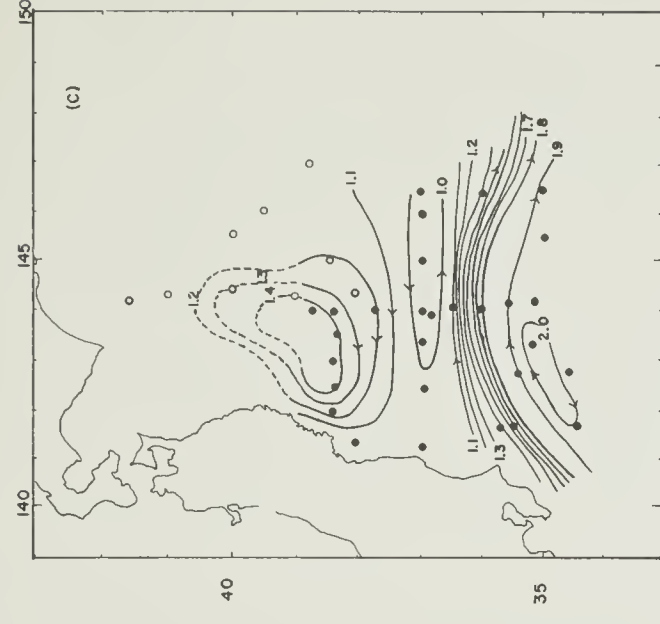
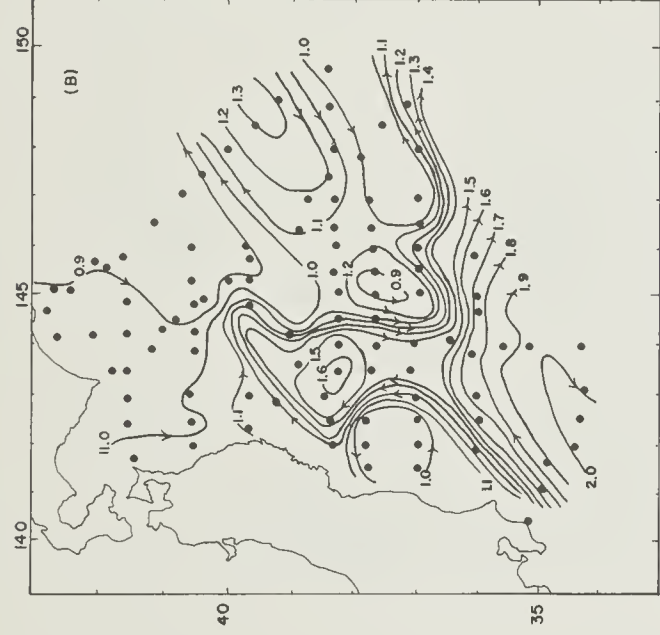
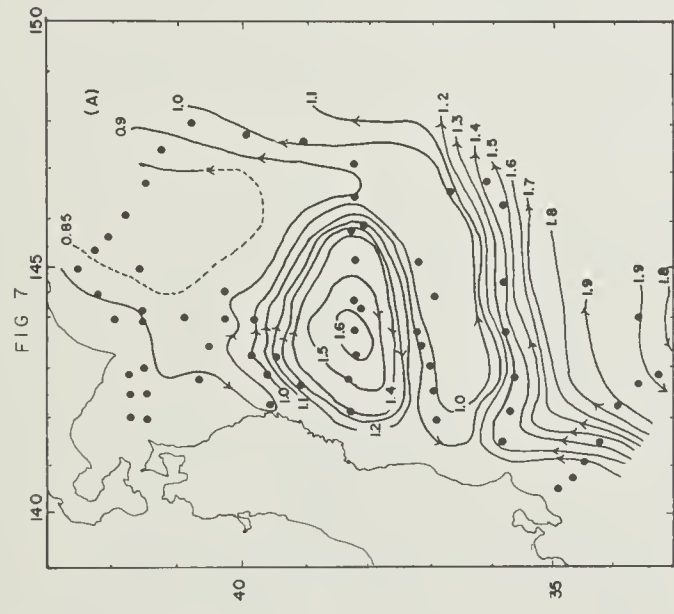
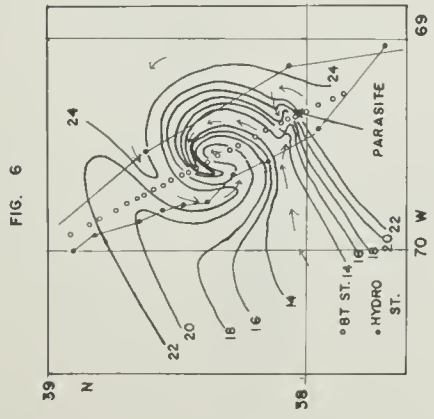
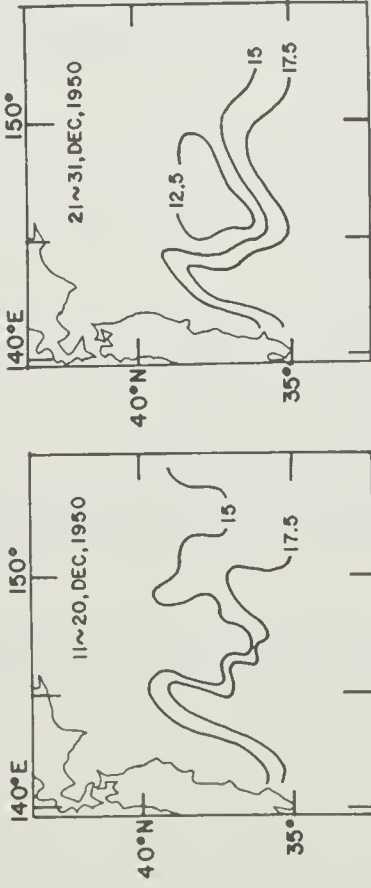
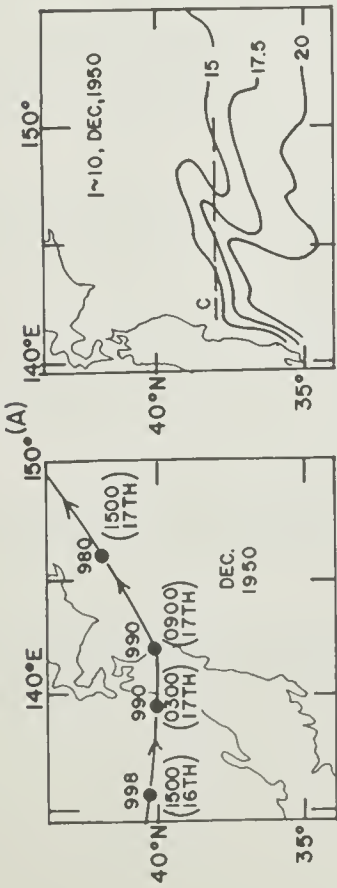




FIG. 8



(B)

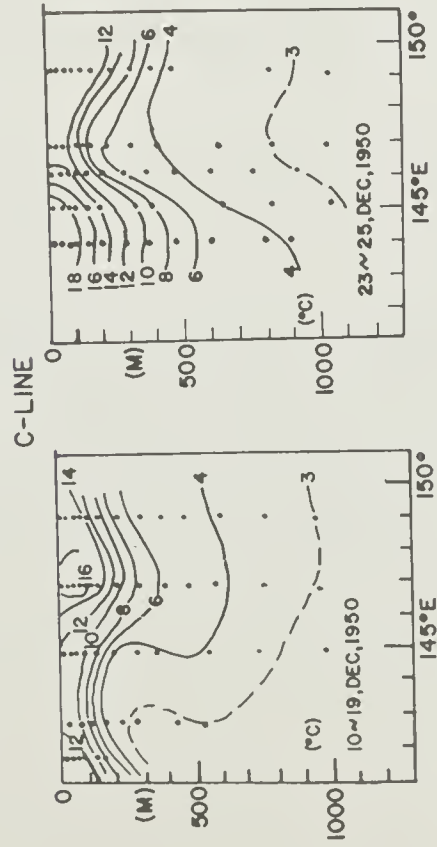
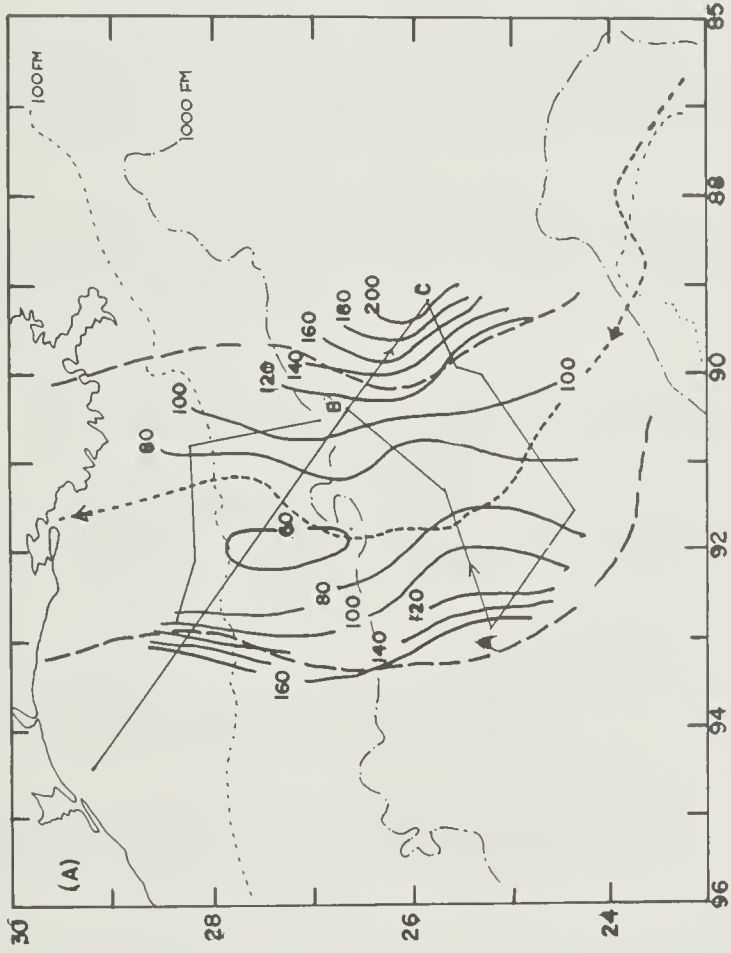


FIG. 9



(B)

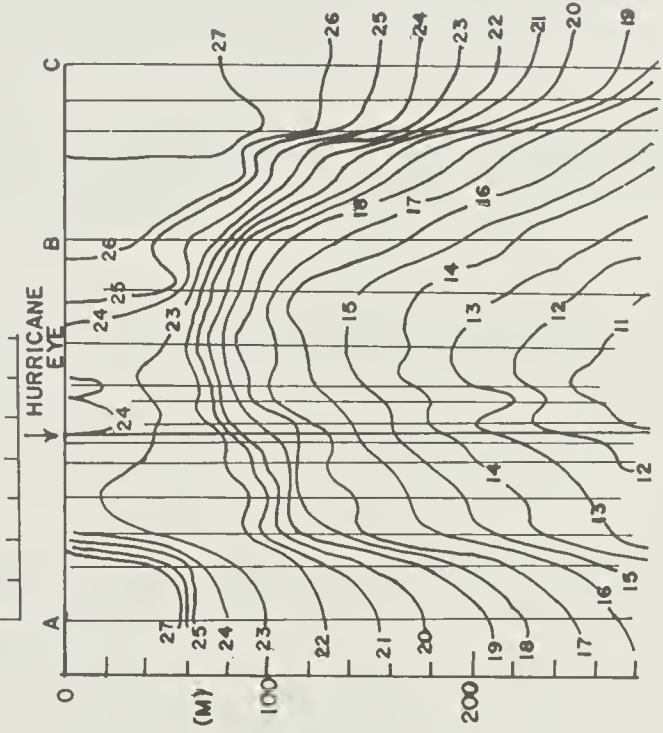
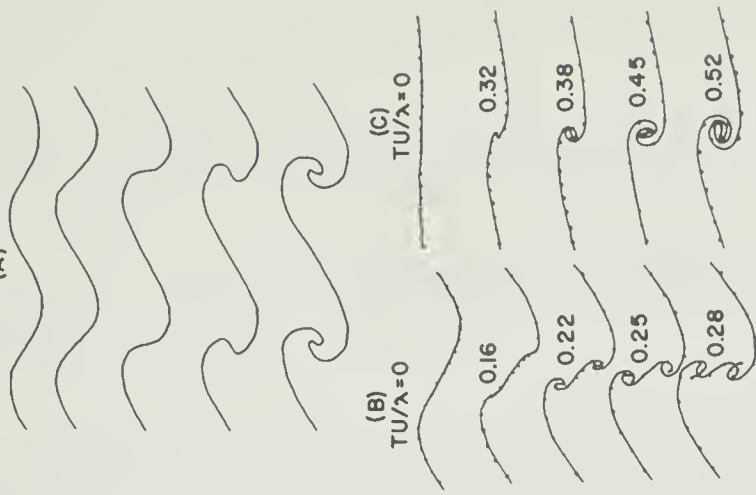


FIG. 10  
(A)







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